3) 11) $(1+y^2)dx+(1+x^2)dy=0$
2022H

$\Rightarrow \frac{1}{1+x^2}dx+\frac{1}{1+y^2}dy=0$

$\therefore \int\frac{1}{1+x^2}dx+\int\frac{1}{1+y^2}dy=C$

$\Rightarrow \tan^{-1}x+\tan^{-1}y=C$ which is reqd solution.

12P 13P 19P1) $(1+e^y)\cos x\,dx+e^y\sin x\,dy=0$

Soln $(1+e^y)\cos x\,dx+e^y\sin x\,dy=0$

$\Rightarrow \frac{\cos x}{\sin x}dx+\frac{e^y}{1+e^y}dy=0$

$\therefore \int\frac{\cos x}{\sin x}dx+\int\frac{e^y}{1+e^y}dy=\log C$

$\Rightarrow \log(\sin x)+\log(1+e^y)=\log C$

$\Rightarrow \log\{\sin x(1+e^y)\}=\log C$

$\Rightarrow \sin x(1+e^y)=C$ which is reqd solution.

15P

8) ~~viii)~~ $\frac{dy}{dx}=\sqrt{y-x}$

Soln ~~Let $z=y-x$~~

Let $z=\sqrt{y-x}$

or $z^2=y-x$

$\therefore 2z\frac{dz}{dx}=\frac{dy}{dx}-1$

$\Rightarrow \frac{dy}{dx}=1+2z\frac{dz}{dx}$

$\therefore 1+2z\frac{dz}{dx}=z$

$\Rightarrow 2z\frac{dz}{dx}=z-1$

$\Rightarrow 2z\frac{dz}{z-1}=dx$

$\therefore 2\int\frac{z}{z-1}dz=\int dx+C$

$\Rightarrow 2\int\frac{z-1+1}{z-1}dz=x+C$

$\Rightarrow 2\int\left(1+\frac{1}{z-1}\right)dz=x+C$

$\Rightarrow 2\left[z+\log(z-1)\right]=x+C$

$\Rightarrow 2\left[\sqrt{y-x}+\log(\sqrt{y-x}-1)\right]=x+C$

which is reqd solution

10) $\frac{dy}{dx} + 1 = e^{x-y}$

$\boxed{12P}$

Sol^n $\frac{dy}{dx} + 1 = e^{x-y}$

Let $z = x - y$

$\therefore \frac{dz}{dx} = 1 - \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dz}{dx}$

$\therefore 1 - \frac{dz}{dx} + 1 = e^{z}$

$\Rightarrow \frac{dz}{dx} = 2 - e^{z}$

$\Rightarrow \frac{dz}{2-e^{z}} = dx$

$\therefore \int dx + C = \int \frac{dz}{2-e^{z}}$

$\Rightarrow x + C = \int \frac{e^{-z}}{2e^{-z}-1} dz$

$= -\frac{1}{2} \int \frac{-2e^{-z}}{2e^{-z}-1} dz$

$= -\frac{1}{2} \log(2e^{-z}-1)$

$= -\frac{1}{2} \log(2e^{y-x}-1)$

$\therefore x + \frac{1}{2} \log(2e^{y-x}-1) = C$ which is reqd solution.

Examine if the following d. eqn is homogeneous and solve:

$\boxed{19P}$

5) $(x^3+y^3)dx - xy^2 dy = 0$

Sol^n $(x^3+y^3)dx - xy^2dy = 0$

$\Rightarrow (x^3+y^3)dx = xy^2 dy$

$\Rightarrow \frac{dy}{dx} = \frac{x^3+y^3}{xy^2}$ —— (1)

It is a homogeneous d. eqn of first order

Now, Let $y = vx$ and or $\frac{y}{x} = v$

$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes,

$v + x\frac{dv}{dx} = \frac{x^3+v^3x^3}{x.v^2x^2}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^3}{v^2} - v$

$$= \frac{1+v^3-v^3}{v^2}$$

$$= \frac{1}{v^2}$$

$$\Rightarrow v^2 dv = \frac{dx}{x}$$

$$\therefore \int w^2 dv = \int \frac{dx}{x} + c$$

$$\Rightarrow \frac{v^3}{3} = \log x + C$$

$$\Rightarrow \frac{1}{3} \cdot \frac{y^3}{x^3} = \log x + C$$

$\Rightarrow y^3 = 3x^3 \log x + 3Cx^3$ which is reqd solution.

13P 7) $(x+y)\,dy + (x-y)\,dx = 0$

Soln $(x+y)\,dy + (x-y)\,dx = 0$

$$\Rightarrow (x+y)\,dy = (y-x)\,dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-x}{x+y} \quad \text{——— ①}$$

It is homogeneous d. equation of 1st order.

Let $y = vx$ or $\frac{y}{x} = v$

$$\therefore \frac{dy}{dx} = \cancel{vx - x}\; v + x\frac{dv}{dx}$$

$\therefore$ ① becomes,

$$v + x\frac{dv}{dx} = \frac{vx - x}{x + vx}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-1}{v+1} - v$$

$$= \frac{v-1-v-v^2}{1+v}$$

$$\Rightarrow \frac{1+v}{1+v^2}\,dv = -\frac{dx}{x}$$

$$\therefore \int \frac{1+v}{1+v^2}\,dv + \int \frac{dx}{x} = C$$

$$\Rightarrow \int \frac{1}{1+v^2}\,dv + \frac{1}{2}\int \frac{2v}{1+v^2} + \log x = C$$

$$\Rightarrow \tan^{-1} v + \frac{1}{2}\log(1+v^2) + \log x = C$$

$$\Rightarrow \tan^{-1}\frac{y}{x} + \frac{1}{2}\log\left(1+\frac{y^2}{x^2}\right) + \log x = C$$

$$\Rightarrow 2\tan^{-1}\frac{y}{x} + \log\left(\frac{x^2+y^2}{x^2}\right) + 2\log x = 2C$$

$\Rightarrow 2\tan^{-1}\frac{y}{x} + \log(x^2+y^2) - \log x^2 + \log x^2 = 2C$

$\Rightarrow 2\tan^{-1}\frac{y}{x} + \log(x^2+y^2) = 2C$ which is reqd solution

D. equations reducable to homogeneous d. equations:-

Ex. Solve. $(6x-5y+4)dy + (y-2x-1)dx = 0$

[2022]

Soln $(6x-5y+4)dy + (y-2x-1)dx = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{2x-y+1}{6x-5y+4} \quad \text{——— (1)}$$

Let Here $\frac{a_1}{a_2} = \frac{1}{3} \neq \frac{b_1}{b_2} = \frac{1}{5}$ (say); $\left(\frac{a_1x+b_1y+c}{a_2x+b_2y+c}\right)$

Let $x = X+h$, $y = Y+K$

$$\therefore \frac{dy}{dx} = \frac{dY}{dX}$$

(1) becomes,

$$\frac{dY}{dX} = \frac{2(X+h)-(Y+K)+1}{6(X+h)-5(Y+k)+4}$$

$$= \frac{2X-Y+2h-k+1}{6X-5Y+6h-5k+4} \quad \text{——— (2)}$$

We choose h & k such that

$$2h-k+1 = 0 \quad \text{——— (3)}$$
$$6h-5k+4 = 0 \quad \text{——— (4)}$$

From (3) & (4) we get

$$\frac{h}{-4+5} = \frac{k}{6-8} = \frac{1}{-10+6}$$

$$\Rightarrow h = \frac{k}{-2} = \frac{1}{-4}$$

$$\therefore h = -\frac{1}{4}, \quad k = \frac{1}{2}$$

With this values of h & k ; (2) becomes

$$\frac{dY}{dX} = \frac{2X-Y}{6X-5Y} \quad \text{——— (5)}$$ It is a homogeneous d.eqn

Let $Y = VX$ or $V = \frac{Y}{X}$

$$\therefore \frac{dY}{dX} = V + X\frac{dV}{dX}$$

$$\therefore\ v + x\frac{dv}{dx} = \frac{2x - vx}{6x - 5vx}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2-v}{6-5v} - v$$

$$= \frac{2 - v - 6v + 5v^2}{6-5v}$$

$$= \frac{5v^2 - 7v + 2}{6-5v}$$

$$\therefore\ \frac{6-5v}{5v^2-7v+2}\,dv = \frac{dx}{x}$$

$$\Rightarrow \frac{5v-6}{5v^2-7v+2}\,dv + \frac{dx}{x} = 0$$

$$\therefore \int \frac{5v-6}{5v^2-7v+2}\,dv + \int \frac{dx}{x} = c$$

$$\Rightarrow \int \frac{\frac{1}{2}(10v-7) + \frac{7}{2} - 6}{5v^2-7v+2}\,dv + \int \frac{dx}{x} = c$$

$$\Rightarrow \frac{1}{2}\int \frac{10v-7}{5v^2-7v+2}\,dv - \frac{5}{2}\int \frac{dv}{5v^2-7v+2} + \log x = c$$

$$\Rightarrow \frac{1}{2}\log(5v^2-7v+2) - \frac{5}{10}\int \frac{dv}{v^2 - \frac{7}{5}v + \frac{2}{5}} + \log x = c$$

$$\Rightarrow \frac{1}{2}\log(5v^2-7v+2) - \frac{1}{2}\int \frac{dv}{\left(v-\frac{7}{10}\right)^2 + \frac{2}{5} - \frac{49}{100}} + \log x = c$$

$$\Rightarrow \frac{1}{2}\log(5v^2-7v+2) - \frac{1}{2}\int \frac{dv}{\left(v-\frac{7}{10}\right)^2 - \left(\frac{3}{10}\right)^2} + \log x = c$$

$$\Rightarrow \frac{1}{2}\log(5v^2-7v+2) - \frac{1}{2}\cdot\frac{1}{2\cdot\frac{3}{10}}\log\frac{v - \frac{7}{10} - \frac{3}{10}}{v - \frac{7}{10} + \frac{3}{10}} + \log x = c$$

$$\Rightarrow \frac{1}{2}\log(5v^2-7v+2) - \frac{5}{6}\log\frac{v-1}{v-\frac{2}{5}} + \log x = c$$

$$\Rightarrow \frac{1}{2}\log(5v^2-7v+2) - \frac{5}{6}\log\frac{5v-5}{5v-2} + \log x = c$$

$$\Rightarrow \frac{1}{2}\log\left(5\frac{y^2}{x^2} - 7\cdot\frac{y}{x} + 2\right) - \frac{5}{6}\log\frac{5\,y/x - 5}{5\,y/x - 2} + \log x =$$

$$\Rightarrow \frac{1}{2}\log\left[5\cdot\frac{(y-1/2)^2}{(x+1/4)^2} - 7\cdot\frac{y-1/2}{x+\frac{1}{4}} + 2\right]$$

$$- \frac{5}{6}\log\left\{\frac{5\left(y-\frac{1}{2}\right) - 5\left(x+\frac{1}{4}\right)}{5\left(y-\frac{1}{2}\right) - 2\left(x+\frac{1}{4}\right)}\right\} + \log\left(x+\frac{1}{4}\right) = c$$